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An Eco-epidemiological Model for Panama Wilt Disease of Banana with Cultural Controls

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ABSTRACT

In this paper an eco-epidemiological model is constructed for the Panama wilt of banana with cultural controls. The model is formed based on five populations which include the non-diseased population. The existence and uniqueness of the model is analyzed. The disease-free and endemic equilibrium points are found. Using the next-generation matrix the reproduction number is calculated. The Local stability of the model is analyzed. Numerical simulation is carried out by using MATLAB.

Keywords— Panama wilt, Equilibrium, Stability, Reproduction number, Bifurcation.

I. Introduction

The primary challenge of the twenty-first century is global food production. Viruses, fungi, diseases, weeds, and pests are destroying the crops[1][2]. Approximately 20%-40% of global crop output is destroyed each year. Plant protection is a critical step in increasing food production and agriculture. Furthermore, improved techniques should be implemented in order to manage various plant diseases that are ecologically sustainable, dependable, socially acceptable and free of hazardous chemicals[13]. For example, crop rotation, biological control, use of resistant varieties and botanical pesticides (like ginger essential oils) can be implemented. Bananas are the most well-known plant and one of the most profitable crops. Banana is widely grown in the world's warm tropical regions, particularly in Brazil, Ecuador, China, the Philippines, Indonesia, Costa Rica, Mexico, Thailand, Colombia and India. It is grown in either a mono-culture or a mixed cropping system[3].

Musa paradisiaca is the scientific name for bananas, which belong to the Musaceae family. Banana fruits and plantains are eaten as desserts and cooked as several dishes all over the world. Viruses, bacteria, fungi and pests all attack banana plants. Fusarium wilt also known as Panama wilt, is one of the most devastating soil-borne diseases[14]. The pathogen Fusarium oxysporum (f. sp. cubense (Foc)) causes Panama wilt[15]. Tropical race (TR4) bananas are susceptible to Fusarium wilt unlike plantain, cooking bananas or a variety of

dessert banana varieties[11]. Panama wilt has two types of symptoms: (i) yellowing of the leaves: this is the most common symptom among banana plants and it affects the older leaves in the beginning. Sometimes later it may be confused with potassium deficiency then it spreads to the immature leaves. After affecting all the leaves the plant looks like a skirt covered with brown leaves, (ii) Green leaf syndrome: This is the opposite of the yellowing of leaves.

Fig. 1 Healthy Plant

The plant will appear with green leaves. This doesn't show whether it is affected or not until all the leaves are bent down and collapsed [11]. Once the Panama wilt is affected, there is no cure until the plant dies. Thus the affected plants produce many infected suckers which doesn't show any symptoms in the fruits but produce less number of fruits with low quality. The figures 1 and 2 show the healthy plant and Panama wilt.

Fig. 2. Panama wilt

Fig. 3. Yellowing of Leaves

Many authors studied and conducted surveys about banana trees, their diseases and controls. M. Chillet et.al explained how the Sigatoka disease deduces the green life of a banana[4]. B.Nannyonga et.al studied the use of contaminated tools, which increase the disease by the Runge-Kutta fourth-order algorithm[13]. Juliet Nakakawa et.al built a model with vertical transmission and inflorescence infection in banana trees and built a model for banana Xanthomonas Wilt which includes de-budding and roguing as controls in a mixed cultivar plantation[8][9]. Kweynaga Eliab Horub et.al investigated the spread of the Xanthomonas Wilt of bananas in East and Central Africa using the host-vector model[10] and Eliab Horub J et.al constructed a mathematical model for the disease Xanthomonas of bananas with asymptotic and symptomatic stages of the disease[5]. Elizabeth Alvarez et.al studied the Black Sigatoka in bananas and explained the disease cycle[6]. John Joel Mapinda et.al explained a method with neglected control techniques in the banana tree[7]. In the following sections, the model for the Panama wilt of bananas is constructed and analyzed.

I. MATHEMATICAL MODEL

The model is constructed by using Ordinary Differential Equations. The model explains that the susceptible(S) population includes all the possibilities of the planting and the suckers population. Susceptible gets infection which has two stages: Initial stage I_0 and Severe stage I_1. Some susceptible population will not get the infection due to the resistance which leads to the high and healthy banana fruits. In the model, the suckers(π) have two possibilities: (1). Straight away it produces healthy and high fruits(ω_3), (2). It gets an infection and produces either low(ω_2) or unhealthy fruits(ω_1). But if the

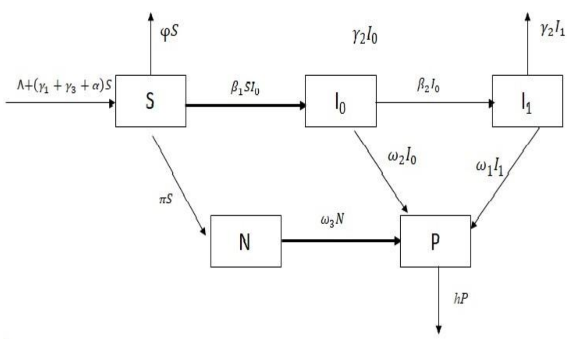


Fig. 4. Diagrammatic Representation of the model

susceptible population gets initial infection I_0 then it produces low banana fruits when it does not get the severe infection I_1. If the initial infection I_0 increases then it leads to the severe infection I_1 and produces low banana fruits and unhealthy fruits. Here, the cultural controls help in the beginning if the field is with proper drainage, inter-cropping of the crops and removal of the infected once after the infection stage.

Thus, the system of Ordinary Differential Equations are as follows:

$$\frac{dS}{dt} = \Lambda + (\gamma_1 + \gamma_3 + \alpha)S - \phi S - \pi S - \beta_1 S I_0$$
$$\frac{dI_0}{dt} = \beta S I_0 - \omega_2 I_0 - \beta_2 I_0 - \gamma_2 I_0$$
$$\frac{dI_1}{dt} = \beta I_0 - (\omega_1 + \gamma_2) I_1$$
$$\frac{dND}{dt} = \pi S - \omega_3 P$$
$$\frac{dP}{dt} = \omega_1 I_1 + \omega_2 I_0 + \omega_3 ND - hP$$

with the initial conditions $S(0)=S_0>0$, $I_0(0)=I_{00}>0$, $I_1(0)=I_{10}>0$, $ND(0)=ND_0>0$ *and* $P(0)=P_0>0$.

Parameter	*Description*
Λ	Planted Seedling rate
γ_1	Proper drainage
γ_2	Removing the infected crop
γ_3	Intercropping, to avoid and reduce infection
α	Suckers rate
ϕ	Degeneration rate
π	Crop rate which is not affected
β_1	Initial infection of disease
β_2	Severe infection of disease
ω_1	Unhealthy production of fruits
ω_2	Low production of fruits
ω_3	High and healthy production of fruits
h	Total Production of fruits

Table .1. Nomenclature of the Model

EXISTENCE AND UNIQUENESS OF THE MODEL

Theorem 3.1. *Let $Q(X)$ be a function where $X=(S, I_0, I_1, ND, P) \in Q_i(X)$ and $X_0=(S_0, I_{00}, I_{10}, ND_0, P_0) \in Q_i(X)$. If the function $Q(X)$ satisfies the Lipchitz condition*

$$\|Q(X)-Q(\bar{X})\| \le L|X-\bar{X}|, \qquad (2)$$

where L is the Lipchitz constant. Then for each X_0, there exists a unique solution $X(t) \in Q_i(X)$ of the system, $\forall t \ge 0$.

Proof. Consider Q(X) as

$$Q_1(X) = \Lambda + (\gamma_1 + \gamma_3 + \alpha)S - \phi S - \Pi S - \beta_1 S I_0$$
$$Q_2(X) = \beta S I_0 - \omega_2 I_0 - \beta_2 I_0 - \gamma_2 I_0$$
$$Q_3(X) = \beta I_0 - (\omega_1 + \gamma_2) I_1 \qquad (3)$$
$$Q_4(X) = \pi S - \omega_3 P$$
$$Q_5(X) = \omega_1 I_1 + \omega_2 I_0 + \omega_3 ND - hP$$

The equation (3) can be rewritten as

$$
\begin{aligned}
Q_1(X) &= \Lambda + p_1 S - \beta_1 S I_0\\
Q_2(X) &= \beta S I_0 - p_2 I_0\\
Q_3(X) &= \beta I_0 - p_3 I_1\\
Q_4(X) &= \pi S - \omega_3 P\\
Q_5(X) &= \omega_1 I_1 + \omega_2 I_0 + \omega_3 ND - hP
\end{aligned} \qquad (4)
$$

Where

$$
\begin{aligned}
p_1 &= (\gamma_1 + \gamma_3 + \alpha - (\phi + \pi))\\
p_2 &= \omega_2 + \beta_2 + \gamma_2 \quad \text{and}\\
p_3 &= \omega 2 + \gamma 2
\end{aligned} \qquad (5)
$$

$$\| Q(X) - Q(\bar{X}) \| = |\Lambda + (p_1 - \pi)S - p_3 I_1 + \omega_1 I_1 + \omega_2 I_0 - hP - \Lambda - (p_1 - \pi)\bar{S} - (p_3 + \omega_1)\bar{I_1} - \omega_2 \bar{I_0} - h\bar{P}|$$

$$\| Q(X) - Q(\bar{X}) \| \leq \left|(p_1 - \pi)(S - \bar{S}) - (p_3 + \omega_1)(I_1 - \bar{I_1}) - \omega_2(I_0 - \bar{I_0}) - h(P - \bar{P}) + \omega_3(ND - \overline{ND})\right|$$

$$\| Q(X) - Q(\bar{X}) \| \leq L\zeta|(S, I_0, I_1, N, P) - (\bar{S}, \bar{I_0}, \bar{I_1}, \overline{ND}, \bar{P})| \leq L\zeta \left|X - X\square\right|$$

Where $L = Max\{(p_1 - \pi), \omega_2, (p_3 + \omega_1), 1, h\}$ and $\zeta = \{ND - \overline{ND}\}$.

EXISTENCE OF EQUILIBRIA

For the system (1),

(1) There exists a unique disease-free point

$$E_0(S, 0, 0, ND, P) = \left(\frac{\Lambda}{(\phi + \pi - (\gamma_1 + \gamma_3 + \alpha))}, 0, 0, \frac{\pi\,\Lambda}{\omega_3\,(\phi + \pi - (\gamma_1 + \gamma_3 + \alpha))}, \frac{\pi\,\Lambda}{h\,(\phi + \pi - (\gamma_1 + \gamma_3 + \alpha))}\right)$$

where S, ND and P are positive only if the condition is satisfied $(\gamma_1 + \gamma_3 + \alpha) + \Lambda > \phi + \pi$

(2) There exists a unique endemic equilibrium

$$E^*(S, I_0^*, I_1^*, ND^*, P^*) = \left(\frac{\omega_2 + \beta_2 + \gamma_2}{\beta_1}, \frac{\Lambda}{\omega_2 + \beta_2 + \gamma_2} + \frac{\gamma_1 + \gamma_3 + \alpha - (\phi + \pi)}{\beta_1}, \frac{\beta_2 I_0^*}{\omega_1 + \gamma_2}, \frac{\pi(\omega_2 + \beta_2 + \gamma_2)}{\omega_3 \beta_1}, I_0^*\left(\frac{\omega_1 \beta_1}{\omega_1 \gamma_2} + \omega_2\right) + \frac{\pi(\omega_2 + \beta_2 + \gamma_2)}{\beta_1}\right)$$

. Here I_0^* is positive only if the condition is satisfied $(\gamma_1 + \gamma_3 + \alpha) > \phi + \pi$.

BASIC REPRODUCTION NUMBER

The basic reproduction number is used to observe the secondary infection of the disease. We can determine whether a disease is an epidemic or it dies out by looking at its reproduction number. The Next Generation Matrix is used to determine the basic reproduction number i. e., $R_0 = FV^{-1}$.

$$F = \begin{vmatrix} \beta_1 S & 0 \\ 0 & 0 \end{vmatrix} \quad \text{and}$$

$$V^{-1} = \begin{vmatrix} \frac{1}{(\omega_2 + \beta_2 + \gamma_2)} & \frac{\beta_2}{(\omega_2 + \beta_2 + \gamma_2)(\omega_1 + \gamma_1)} \\ 0 & \frac{1}{\omega_1 + \gamma_1} \end{vmatrix}$$

When substituting the disease-free equilibrium in F we get the R_0 as

$$R_0 = \frac{\Lambda \beta_1}{(\phi + \pi - (\gamma_1 + \gamma_3 + \alpha))(\omega_2 + \beta_2 + \gamma_2)}$$

STABILITY OF THE MODEL

A. LOCAL STABILITY OF THE MODEL

The Jacobin matrix of the system (1) is

$$\begin{pmatrix} \gamma_1 + \gamma_3 + \alpha - (\phi + \pi) - \beta_1 I_0 & -\beta_1 S & 0 & 0 \\ \beta_1 I_0 & \beta 1S - (\omega 2 + \beta 2 + \gamma 2) & 0 & 0 \\ 0 & \beta_2 & -(\omega 1 + \gamma 2) & 0 \\ \pi & 0 & 0 & -\omega_3 \end{pmatrix} \qquad (6)$$

Theorem 6.1. *The system 1 is Locally stable for disease-free equilibrium only if all the eigenvalues are negative.*

Proof: In (8), substituting the disease-free equilibrium points, we get

$$J(DFE) = \begin{pmatrix} \gamma_1 + \gamma_3 + \alpha - (\phi + \pi) & -\beta_1 S & 0 & 0 \\ 0 & \beta_1 S - (\omega_2 + \beta_2 + \gamma_2) & 0 & 0 \\ 0 & \beta_2 & -(\omega_1 + \gamma_2) & 0 \\ \pi & 0 & 0 & -\omega_3 \end{pmatrix}$$

The characteristic equation is

$$[\gamma_1 + \gamma_3 + \alpha - (\phi + \pi) - \lambda)(\beta_1 S - (\omega_2 + \beta_2 + \gamma_2) - \lambda)(-(\omega_1 + \gamma_2) - \lambda)(-\omega_3 - \lambda) = 0 \qquad (8)$$

$$\Rightarrow \lambda_1 = -((\phi + \pi) - (\gamma_1 + \gamma_3 + \alpha)),$$
$$\lambda_2 = -(\omega_2 + \beta_2 + \gamma_2) - \frac{\beta_1 \Lambda}{\gamma_1 + \gamma_3 + \alpha - (\phi + \pi)},$$
$\lambda_3 = -(\omega_1 + \gamma_2)$ and $\lambda_4 = -\omega_3$.

Since all the eigenvalues are negative. The system is stable at disease-free equilibrium.

Theorem 6.2. *The system 1 is Locally stable for endemic equilibrium only if*

$$\frac{\beta_1 \Lambda}{\phi + \pi - (\gamma_1 + \gamma_3 + \alpha)} < \omega_2 + \beta_2 + \gamma_2$$

Proof. In (6), Substituting the endemic equilibrium, we get

$$J(EE) = \begin{pmatrix} \gamma_1 + \gamma_3 + \alpha - (\phi + \pi) - \beta_1 I_0 & -\beta_1 S & 0 & 0 \\ \beta_1 I_0 & \beta 1S - (\omega 2 + \beta 2 + \gamma 2) & 0 & 0 \\ 0 & \beta_2 & -(\omega 1 + \gamma 2) & 0 \\ \pi & 0 & 0 & -\omega_3 \end{pmatrix} \qquad (9)$$

The characteristic equation is

$$(\gamma_1 + \gamma_3 + \alpha - (\phi + \pi) - \beta_1 I_0 - \lambda)(\beta_1 S - (\omega_2 + \beta_2 + \gamma_2) - \lambda)(-(\omega_1 + \gamma_2) - \lambda)(-\omega_3 - \lambda) = 0 \qquad (10)$$

$\lambda_3 = -(\omega_1 + \gamma_2)$ and $\lambda_4 = -\omega_3$. The remaining eigenvalues can be obtained from

$$(\gamma_1 + \gamma_3 + \alpha - (\phi + \pi) - \beta_1 I_0 - \lambda)(\beta_1 S - (\omega_2 + \beta_2 + \gamma_2) - \lambda) = (\gamma_1 + \gamma_3 + \alpha - (\phi + \pi))\beta_1 S - (\gamma_1 + \gamma_3 + \alpha - (\phi + \pi))(\omega_2 + \beta_2 + \gamma$$
$$\beta_1 I_0(\omega_2 + \beta_2 + \gamma_2) - \beta 1\, I_0 \lambda - \beta_1 S\, \lambda - (\omega_2 + \beta_2 +$$
$$\lambda_2 + Q_0 \lambda + Q_1 = 0 \qquad (11)$$

Here

$Q_0=\omega_2+\beta_2+\gamma_2-\beta_1 S+\beta_1 I_0+(\phi+\pi)-(\gamma_1+\gamma_3+\alpha)$ and $Q_1=(\gamma_1+\gamma_3+\alpha-(\phi+\pi))\beta_1 S-(\gamma_1+\gamma_3+\alpha-(\phi+\pi))(\omega_2+\beta_2+\gamma_2)+\beta_1 I_0(\omega_2+\beta_2+\gamma_2)$.

By Routh-Hurwitz criteria $Q_0 > 0$ if $\frac{\beta_1\Lambda}{\phi+\pi-(\gamma_1+\gamma_3+\alpha)} < \omega_2+\beta_2+\gamma_2$ and $Q_1 > 0$ only if $R_0 > 1$. Thus, the system is locally stable at endemic equilibrium.

Theorem 6.3. *If $a<0$, $b>0$ for $\beta_1^* = \frac{(\phi+\pi-(\gamma_1+\gamma_3+\alpha))(\omega_2+\beta_2+\gamma_2)}{\Lambda}$, then the system exhibit transcritical bifurcation.*

Proof. To verify the stability using $R_0 = 1$, we apply center manifold theory. Considering $(S, I_0, I_1) = (x_1, x_2, x_3)$ and $F(x)=\frac{dx}{dt}$ with $F(x)=(f_1, f_2, f_3)^T$. The model is rewritten in the form of

$$\frac{dx_1}{dt}=\Lambda + (\gamma_1+\gamma_3+\alpha)x_1 - \phi\, x_1 - \Pi\, x_1 - \beta_1\, x_1 x_2$$
$$\frac{dx_2}{dt}=\beta\, x_1\, x_2 - \omega_2\, x_2 - \beta_2\, x_2 - \gamma_2\, x_2 \qquad (11)$$
$$\frac{dx_3}{dt}=\beta\, x_2 - (\omega_1+\gamma_2)\, x_3$$

When $R_0= 1$, the transmission rate is taken as

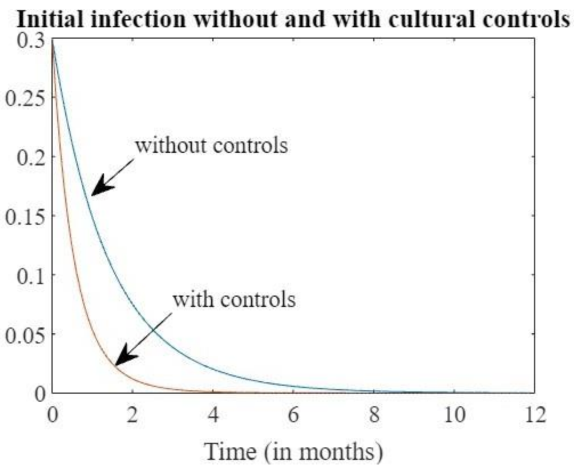
$\beta_1^* = \frac{(\phi+\pi-(\gamma_1+\gamma_3+\alpha))(\omega_2+\beta_2+\gamma_2)}{\Lambda}$. The Jacobian of the system is

$$J=\begin{pmatrix}(\gamma_1+\gamma_3+\alpha)-(\phi+\pi) & -\beta_1 x_1 & 0\\ \beta_1 x_2 & \beta_1 x_1-(\omega_2+\beta_2+\gamma_2) & 0\\ 0 & \beta_2 & -(\omega_1+\gamma_2)\end{pmatrix} \qquad (12)$$

The disease-free equilibrium $x_1=\frac{\Lambda}{(\phi+\pi-(\gamma_1+\gamma_3+\alpha))}$, $x_2=x_3=0$. Substituting the disease-free equilibrium, we get

$$J=\begin{pmatrix}(\gamma_1+\gamma_3+\alpha)-(\phi+\pi) & \frac{-\Lambda\beta_1}{(\phi+\pi-(\gamma_1+\gamma_3+\alpha))} & 0\\ 0 & \frac{\beta_1\Lambda}{(\phi+\pi-(\gamma_1+\gamma_3+\alpha))}-(\omega_2+\beta_2+\gamma_2) & 0\\ 0 & \beta_2 & -(\omega_1+\gamma_2)\end{pmatrix} \qquad (13)$$

To find the right eigenvectors $Ju = 0$ where $u = (u_1, u_2, u_3)^T$, we attain



$$(\gamma_1+\gamma_3+\alpha)-(\phi+\pi)u_1-\frac{\Lambda\beta_1}{(\phi+\pi-(\gamma_1+\gamma_3+\alpha))}u_2=0$$
$$\frac{\beta_1\Lambda}{(\phi+\pi-(\gamma_1+\gamma_3+\alpha))}u_2-(\omega_2+\beta_2+\gamma_2)u_2=0$$
$$\beta_2 x_2-(\omega_1+\gamma_2)x_3=0 \qquad (14)$$

From (14), the values are $u_1=\frac{-\Lambda\beta_1}{(\phi+\pi-(\gamma_1+\gamma_3+\alpha))^2}u_2$, $u_2=u_2$ and $u_3=\frac{\beta_2}{\omega_1+\gamma_2}$. To find the left eigen vectors $vJ = 0$, where $v = (v_1, v_2, v_3)$, we get

$$(\gamma_1+\gamma_3+\alpha)-(\phi+\pi)v_1=0$$
$$-\frac{\Lambda\beta_1}{(\phi+\pi-(\gamma_1+\gamma_3+\alpha))}v_2+\frac{\beta_1\Lambda}{(\phi+\pi-(\gamma_1+\gamma_3+\alpha))}v_2-(\omega_2+\beta_2+\gamma_2)v_2+\beta_2 v_2=0$$
$$-(\omega_1+\gamma_2)v\_3=0 \qquad (15)$$

From (15) we have $v_1 = v_3 = 0$ and v_2 is calculated in such a way that it satisfies the conditions $v.u = 1$. The only non-zero partial derivatives of f_2 are

$$f_2 = \beta_1 x_1 x_2 - (\omega_2+\beta_2+\gamma_2)x_2$$

$$\frac{\partial^2 f_2}{x_1\, x_2}=\beta_1^*=\frac{\partial^2 f_2}{x_2\, x_1},$$
$$\frac{\partial^2 f_2}{x_2\, \beta_1}=x_1=\frac{\Lambda}{(\phi+\pi-(\gamma_1+\gamma_3+\alpha))}$$

All the other partial derivatives of f_2 are zero. To find the bifurcation's direction at $R_0 = 1$, we need to determine the signs of a and b, where a and b are bifurcation coefficients. The values of 'a' and 'b' at β_1^* are given below:

$$a=v_2 u_1 u_2\frac{\partial^2 f_2}{x_1\, x_2}=\frac{-\Lambda\beta_1^*}{(\phi+\pi-(\gamma_1+\gamma_3+\alpha))}<0$$

$$b=v_2 u_2\frac{\partial^2 f_2}{x_2\, \beta_1}=\frac{\Lambda}{\left(\phi+\pi-(\gamma_1+\gamma_3+\alpha)\right)}>0$$

Hence $a < 0$ and $b > 0$, this shows forward bifurcation, i.e., the system undergoes transcritical bifurcation at $R_0= 1$.

NUMERICAL SIMULATION

The parameter values are calculated from the survey which was undertaken in Tiruvannamalai. The parameters are Λ=0.9, γ_1=0 or 0.5, γ_2=0 or 1, γ_3=0 or 0.5, α=0.05, ϕ=0.01, π=0.9, β_1=0.5, β_2=0.8, ω_1=0.2, ω_2=0.3, ω_3=0.11, h=0.6, N(Total number of plants)=1000, S=900, $I_0 = 300$, $I_1 = 500$, ND=900 and R=500.

Figure 5: Initial infection population

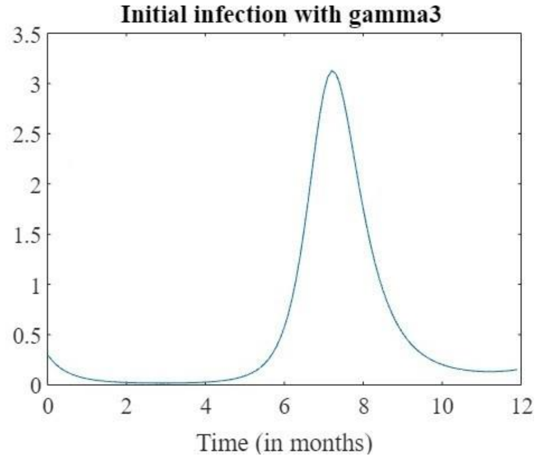


Figure 6: Initial infection with the control intercropping γ_3

Figure 5 displays the initial infection population. The graph given above exposes the impact of cultural control γ_1 and γ_2 are effective in reducing the population of the initial infection whereas Figure 6 shows that the cultural control γ_3 which explains that intercropping increases the initial infection in the plant population.

Figure 7 depicts the graph of severe infection population which shows that the cultural controls γ_1 and γ_2 helps in decreasing the severe infection population but γ_3 increases the severe infection population after 6 months of growth which is shown in the figure 8. Figure 9 represents the non-diseased population with cultural controls γ_1 and γ_2 where the graph depicts that if the plants didn't get any infection then cultural controls γ_1 and γ_3 will increase the non-diseased population.

Figure 10 explains that the cultural control γ_3 increases the non-diseased population. Figure 11 exhibits the graph of the production of bananas, while using the cultural controls γ_1 and γ_2 the production of bananas is increased. Figure 12 presents the graph where γ_3 also helps in the increase of production.

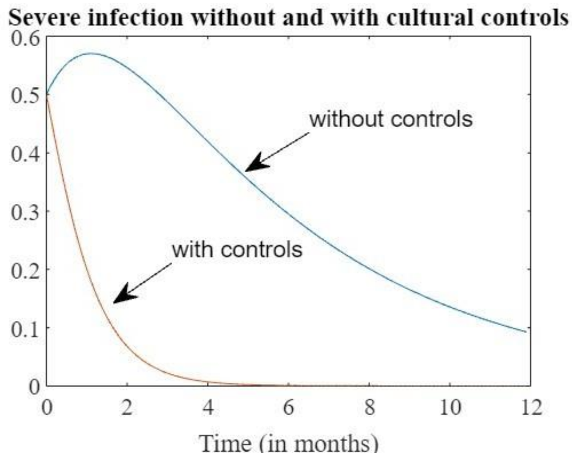


Figure 7: Severe infection population

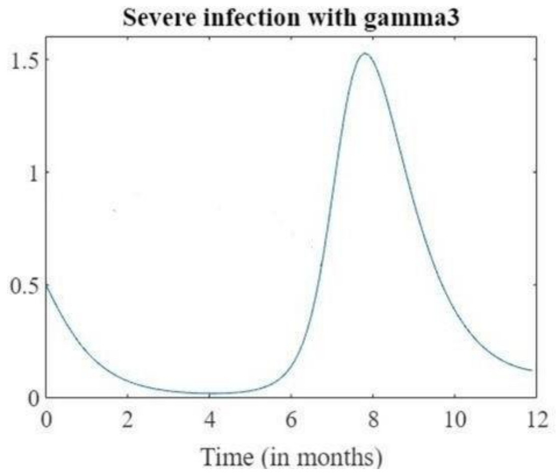


Figure 8: Severe infection with the control intercropping γ_3

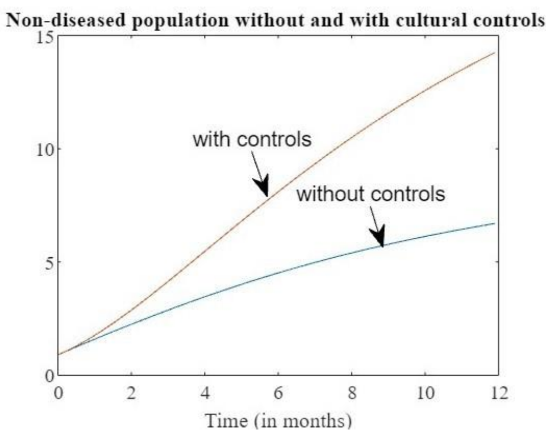


Figure 9: Non-diseased population

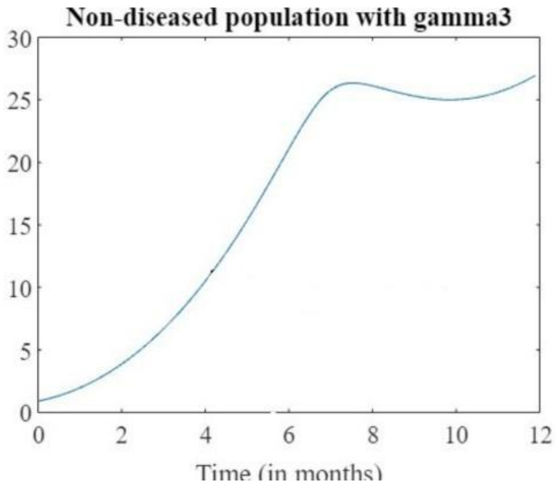


Figure 10: Non-diseased with intercropping γ_3

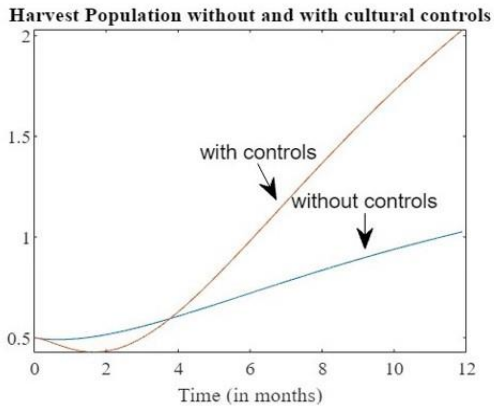


Figure 11: Production population

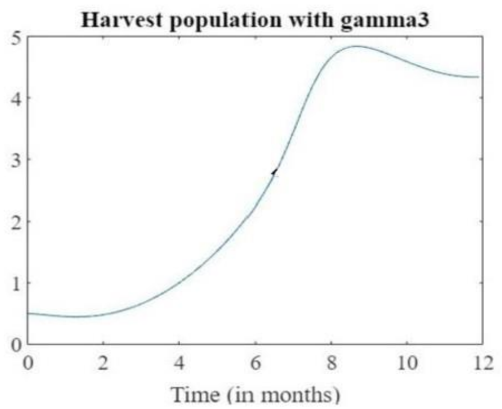


Figure 12: Production with intercropping γ_3

CONCLUSION

The analysis of the eco-epidemiological model for Panama wilt of bananas with cultural controls concludes as follows. The existence and uniqueness are executed in this model. The equilibria: disease-free and endemic equilibrium are found. The reproduction number is calculated by the next-generation matrix. The Local stability of the model is analyzed for the following three cases: case(i)if all the eigenvalues are negative the system is stable for disease-free equilibrium. i.e, Since all the eigenvalues which express that the disease doesn't spread in the plants, case(ii) if $\frac{\Lambda}{(\phi+\pi-(\gamma_1+\gamma_2+\alpha))} > (\omega_2+\beta_2+\gamma_2)$, the system is stable for endemic equilibrium. i.e., If the infection rate in the planted population is higher than the reduced fruit output and the removal of diseased ones, the disease will spread throughout the plant population. (iii) If a<0, b>0 for $\beta_1^{*}=\frac{(\phi+\pi-(\gamma_1+\gamma_3+\alpha))(\omega_2+\beta_2+\gamma_2)}{\Lambda}$, then system exhibits transcritical bifurcation. Thus, the paper explains how cultural controls are useful in the Panama wilt of banana. The cultural controls are γ_1(Drainage), γ_2(Removal of infected) and γ_3(Intercropping), in this model the two cultural controls γ_1(Drainage) and γ_2(Removal of infected) helps in not spreading the disease in two state variables,i.e., Initial infection and Non-diseased population. But the cultural control γ_3(Intercropping) increases the diseasepopulation in Severe infection. Usage of cultural controls increases the production population which is represented in the given graphs. Hence, this model helps in reducing the disease spread using cultural controls.

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